JOURNAL OF COMPUTATIONAL PHYSICS 28, 437–438 (1978)

Letter to the Editor

An Artificial Viscosity for Two-Dimensional Hydrodynamics

Nearly all two-dimensional hydrodynamics computations are subject to short-wavelength numerical instabilities. To suppress the growth of the nonphysical disturbances, some sort of artificial viscosity is usually required.

A recent paper by Chan [1] describes an improved damping method, which was used in a mixed Eulerian-Lagrangian hydrodynamics computer program. This method eliminates the short-wavelength "alternating instability", while at the same time dissipating a relatively small amount of kinetic energy. The purpose of the present note is to show that this method is really a nine-point artificial viscosity, and to show why it is more effective than the more usual five-point form.

Consider a quantity u_{ij} which is defined at the vertices of a two dimensional mesh. Chan defines the "discrepancy parameter" ψ for the cell centered at the point $(i + 1/2, j + 1/2)$ as

$$\psi = \psi_{i+1/2,j+1/2} = (u_{i+1,j+1} + u_{ji}) - (u_{i+1,j} + u_{i,j+1}).$$

This quantity is designed to be sensitive to "alternating errors", perturbations which alternate in sign along adjacent diagonals of the mesh. The values of u_{ij} at the vertices of the cell are adjusted according to the replacements:

$$\begin{aligned} u_{ij} &= u_{ij} - k\psi/4, \\ u_{i+1,j} &= u_{i+1,j} + k\psi/4, \\ u_{i+1,j+1} &= u_{i+1,j+1} - k\psi/4, \\ u_{i,j+1} &= u_{i,j+1} + k\psi/4. \end{aligned}$$

It is easier to see the effect of this correction by examining a particular vertex. Each vertex belongs to four cells, and receives four corrections of the form given above. The total correction to u_{ij} is given by:

$$\begin{aligned} u_{ij} = u_{ij} + k/4[2(&u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}) \\ &-(u_{i+1,j+1} + u_{i+1,j-1} + u_{i-1,j+1} + u_{i-1,j-1}) - 4u_{ij}]. \end{aligned} \tag{1}$$

This formula may be compared to the simpler, more usual expression:

$$u_{ij} = u_{ij} + h/4[u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{ij}]. \tag{2}$$

0021-9991/78/0283–0437$02.00/0

It is readily shown that (1) is superior to (2). Consider a perturbation of the form

$$u_{mn} = ue^{im\theta+in\phi}, \tag{3}$$

where

$$\theta = 2\pi J/M, \qquad \phi = 2\pi K/N$$

and

$$0 \leqslant m \leqslant M, \qquad 0 \leqslant n \leqslant M.$$

Here, J, K are mode numbers; m, n refer to mesh points; and $i = -1^{1/2}$. Substituting (3) into (1) yields

$$u = \left(1 - 4k \sin^2 \frac{\theta}{2} \sin^2 \frac{\phi}{2}\right) u,$$

while substituting into (2) gives

$$u = \left[1 - h\left(\sin^2 \frac{\theta}{2} + \sin^2 \frac{\phi}{2}\right)\right] u.$$

The superiority of (1) is now evident. Both methods can eliminate the shortest wavelength mode, if $k = \frac{1}{4}$ and $h = \frac{1}{2}$. On the other hand, for the longwavelength physical modes clearly (1) introduces less damping. If (2) decreases the longest-wavelength mode by $\epsilon \ll 1$, then (1) only decreases the same mode by ϵ^2.

I have used (1) in my own magnetohydrodynamics code, with excellent results.

Reference

1. R. K.-C. Chan, *J. Computational Physics* **17** (1975), 311.

Received: March 11, 1976

H. R. Strauss

Fusion Research Center
University of Texas at Austin
Austin, Texas 78712